#Real Numbers

Topics.

1. Fundamental Theorem of Arithmetic –
2. Proofs of irrationality of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$

Fundamental theorem of arithmetic

Every integer greater than 1 is either a prime number or can be expressed as a product of prime numbers, and this factorization is unique, except for the order of the prime factors.

Example:- $60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$

Composite number = product of prime numbers

Relation between H.C.F and L.C.M

Any two positive integers 'a' and 'b', the relation between these numbers and their H.C.F and L.C.M is,

$$H.C.F(a,b) \times L.C.M(a,b) = a \times b$$

$$H.C.F(a,b) = \frac{a \times b}{L.C.M(a,b)}$$

$$L.C.M(a,b) = \frac{a \times b}{H.C.F(a,b)}$$

Irrational Numbers.

An Irrational number is a real number that cannot be expressed in the form p/q, where p and q are Integers and $q \neq 0$.

Example:- $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, π (pi), Euler's number.

Note:- Square roots of non-perfect squares are irrational.

Proof that $\sqrt{2}$ is irrational. (V.V.I)

Let $\sqrt{2}$ is rational number.

Then it can be written as $\sqrt{2} = \frac{p}{q}$

Squaring both sides

$2 = \frac{p^2}{q^2}$, where p and q are co-prime Integers.

$\Rightarrow \quad p^2 = 2q^2$, p^2 is divisible by 2 —— (1)

or, 2 divides p

Let $p = 2k$ putting in equation (1)

$(2k)^2 = 2q^2$

$\Rightarrow 4k^2 = 2q^2$

$\Rightarrow \quad q^2 = 2k^2$ —— (2)

So, 2 divides q^2, or, 2 divides q.

Contradiction.

Both p and q are divisible, which contradicts the assumption that they are co-prime.

Conclusion.

$\sqrt{2}$ is irrational number.

Same as $\sqrt{3}$ and $\sqrt{5}$ are Irrational.

—— The End ——