Polynomials

Topics.

(a) zeros of polynomial.

(b) Relation between zeros and co-efficients of quadratic polynomials.

Polynomial (Introduction)

⇓

An algebraic expression of the form

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} \text{----} + a_1 x + a_0,$$

where n = non-negative Integer

(मतलब x का power हमेशा +ve होगा)

and $a_0, a_1, a_2 \text{---} a_n$ are constants.

Also know as Co-efficients of polynomial.

Degree of polynomial

⇓ Highest power of x in a polynomial $p(x)$.

<u>Example</u>: $p(x) = x^3 - 3$

$p(x)$ is 3-degree polynomial.

Reason :- maximum power on x is 3.

$$P(p) = 76P^7 - 3P^5 + \frac{\sqrt{3}}{2}P - 6$$

This is ~~6~~ 7-degree polynomial.

Reason :- maximum power on x is 7.

Types of polynomials

1. Linear polynomial :-

General form $p(x) = ax+b, a \neq 0$

Example: $p(x) = 3x+5$

2. Quadratic polynomial :-

General form $p(x) = ax^2+bx+c, a \neq 0$

Example: $p(x) = 3x^2+5x+6$

Value of polynomial at a Given point.

If $p(x)$ is a polynomial.
find $p(x)$ at $x=\alpha$, where α is a real number.
very simple to find put $x=\alpha$ in $p(x)$ and find $p(\alpha)$.

Example: If $p(x) = 3x^2-5$ then its value at $x=1$ is -

$$p(1) = 3(1)^2-5 = 3\times1-5 = 3-5 = -2$$

Zero of polynomial

A real number 'α' is said to be zero of a polynomial $p(x)$, if $p(\alpha)=0$.

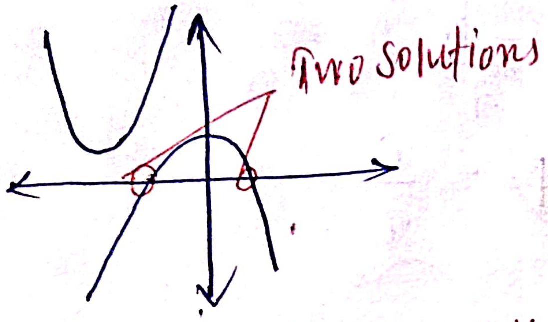
Example: $p(x) = x^2+5x+6$

$$p(-2) = (-2)^2+5(-2)+6 = 4-10+6$$
$$= 10-10 = 0$$

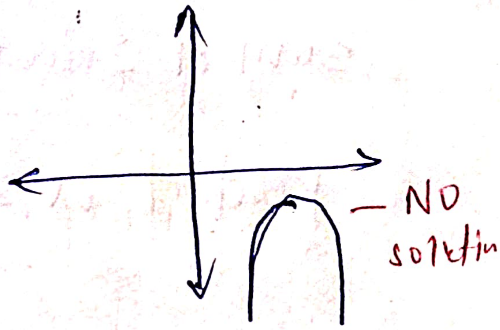
Here $p(-2) = 0$, Hence, $x=-2$ is a zero of polynomial $p(x) = x^2+5x+6$.

Note:- The zeros of a polynomial $p(x)$ are precisely the x-co-ordinates of the points, where the graph of $y = p(x)$ intersects the X-axis.

(a.)



(b.)



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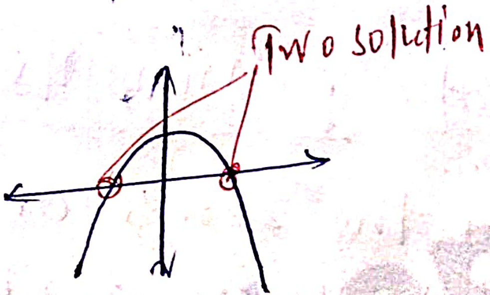

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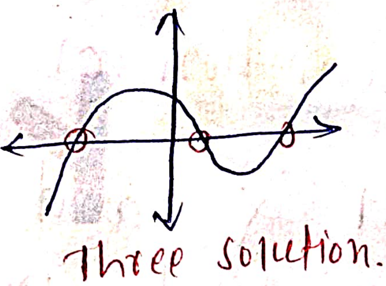

Relation between zeroes and co-efficients of a polynomial.

Quadratic polynomial

General form :- $p(x) = ax^2 + bx + c, \ a \neq 0$

Let 'α' and 'β' are zeroes of the polynomial $p(x)$.

then by factor theorem.

$(x-\alpha)$ and $(x-\beta)$ both are factors of $p(x)$.

Therefore,
$$ax^2 + bx + c = k(x-\alpha)(x-\beta)$$
$$= k[x^2 - x\beta - x\alpha + \alpha\beta]$$
$$= kx^2 - x\beta k - x\alpha k + k\alpha\beta$$
$$= kx^2 - k(\alpha+\beta)x + k\alpha\beta$$

On comparing the co-efficients

$$ax^2+bx+c = kx^2 - k(\alpha+\beta)x + k\alpha\beta$$

$$a = k, \quad \alpha+\beta = -\frac{b}{a} \quad \alpha\beta = \frac{c}{k}$$

sum of zeroes $(\alpha+\beta) = \frac{-b}{a}$

product of zeroes $(\alpha\beta) = \frac{c}{a}$

formation of Quadratic polynomials

If α and β are the zeroes of a quadratic polynomial, then quadratic polynomial will be —

$$p(x) = x^2 - (\text{sum of zeros})x + \text{product of zeros}$$

— The End —