Triangle

#class-10th

⇒ Revision Notes.

⇒ Syllabus.

1. Similar Triangle
2. criteria for similarity of Triangle.

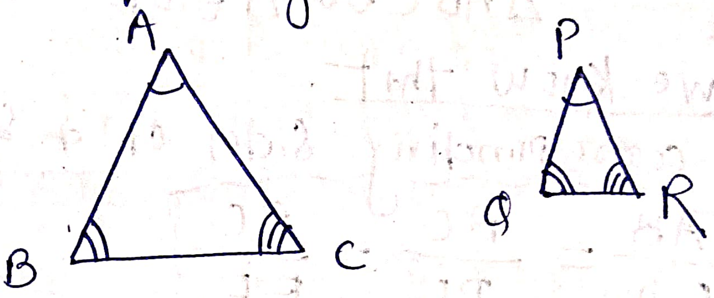
Similar triangle

↳ Two triangle are said to be similar, if

1. their corresponding Angles are equal.
2. their corresponding sides are proportional.

(i.e the ratio of the lengths of corresponding sides are same.)

Example =



ΔABC is similar to ΔPQR, then

$$\boxed{\angle A = \angle P, \quad \angle B = \angle Q, \quad \angle C = \angle R}$$

&

$$\boxed{\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}}$$

Symbolic Representation of Similar triangle.

⇒ $\Delta ABC \sim \Delta PQR$, where $\left.\begin{array}{l}\angle A = \angle P \\ \angle B = \angle Q \\ \angle C = \angle R\end{array}\right\}$

$\left.\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}\right\}$

Note: it will be wrong if we write,

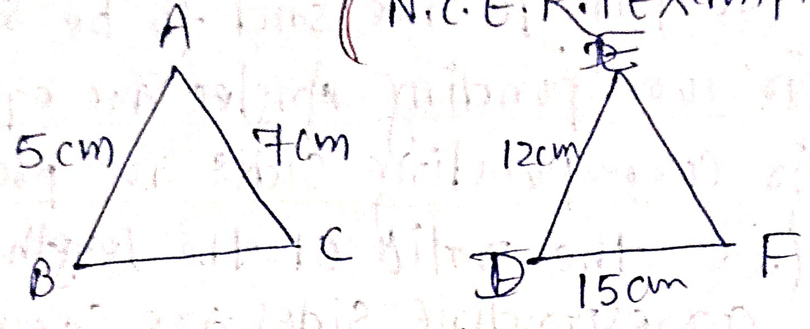
$\Delta ABC \sim \Delta QPR$ ✗

$\Delta ABC \sim \Delta QRP$ ✗

Example Question

(1.) It is given that $\Delta ABC \sim \Delta EDF$ such that $AB = 5$ cm, $AC = 7$ cm, $DF = 15$ cm and $DE = 12$ cm. Find the length of the remaining sides of the triangle.

(N.C.E.R.T Examples)



Given $\Delta ABC \sim \Delta EDF$

we know that corresponding sides of a similar Δ are proportional

$$\frac{AB}{ED} = \frac{BC}{DF} = \frac{AC}{EF}$$

$$\Rightarrow \frac{5}{12} = \frac{BC}{15} = \frac{7}{EF}$$

$$\frac{5}{12} = \frac{7}{EF}$$

$$\Rightarrow 5 \times EF = 84$$

$$\therefore EF = \frac{84}{5} = 16.8 \text{ cm}$$

Ans

$$\frac{5}{12} = \frac{BC}{15}$$

$$\Rightarrow BC = \frac{5 \times \cancel{15}^{5}}{\cancel{12}_{4}} = \frac{25}{4}$$

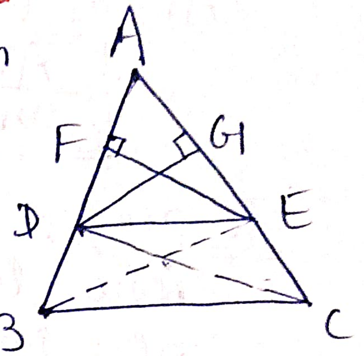
$$= 6.25 \text{ cm}$$

Ans

BPT (Basic proportionality Theorem)

Theorem-1. If a line is drawn parallel to one side of a triangle to Intersect the other two sides in distinct points, then the other two sides are divided in the same Ratio.

Note:- Also called Thales theorem.



Given:- ABC is a triangle in which DE parallel to BC and DE Intersect AB at D and AC at E.

To prove - $\boxed{\frac{AD}{DB} = \frac{AE}{EC}}$

Construction:- join ~~DEA~~ and ~~BE~~, DG ⊥ AE and EF ⊥ AD

proof- In △ADE and In ~~△AED~~ △BDE

$$\frac{\text{area of } \triangle ADE}{\text{area of } \triangle BDE} = \frac{\cancel{\frac{1}{2}} \times AD \times \cancel{EF}}{\cancel{\frac{1}{2}} \times DB \times \cancel{EF}} = \frac{AD}{DB} \quad \text{—— (1)}$$

Similarly, In △AED & △BDE

$$\frac{\text{area of } \triangle AED}{\text{area of } \triangle CDE} = \frac{\cancel{\frac{1}{2}} \times AE \times \cancel{DG}}{\cancel{\frac{1}{2}} \times EC \times \cancel{DG}} = \frac{AE}{EC} \quad \text{—— (2)}$$

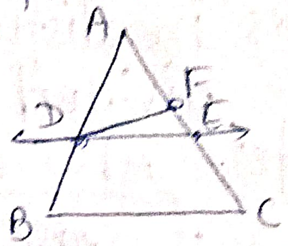
Since △BDE and △CDE stand on the same base DE and between the same parallel line,

area (△BDE) = area of △CDE —— (3)

from Eqn (1) & (2) $\boxed{\frac{AD}{DB} = \frac{AE}{EC}}$ proved

Theorem: 2 Converse of Basic proportionality

Converse of Thale's Theorem - If a line divides any two sides of a triangle in the same ratio then the line must be parallel to the third side.



Proof: In a ΔABC, DE is a line & Intersecting AB at D and AC at E, such that

$$\boxed{\frac{AD}{DB} = \frac{AE}{EC}} \quad \text{--- (1)}$$

To prove - $DE \parallel BC$

Now let us assume that DE is not parallel to BC. Now draw a another line DF which Intersect AC at F and $DF \parallel BC$.

By B.P.T (Basic proportionality theorem)

$$\frac{AD}{DB} = \frac{AF}{FC} \quad \text{--- (2)}$$

from eqn (1) & eqn (2)

$$\frac{AE}{EC} = \frac{AF}{FC}$$

Adding both side 1.

$$\frac{AE}{EC} + 1 = \frac{AF}{FC} + 1$$

$$\Rightarrow \frac{AE+EC}{EC} = \frac{AF+FC}{FC}$$

$$= \frac{AC}{EC} = \frac{AC}{FC}$$

$$\therefore \boxed{EC = FC} \quad [\because \text{All numerator are same}]$$

Thus we can say that point E and F are coincide

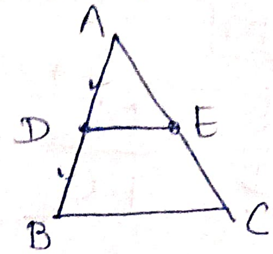
Thus DE is also paraller to BC prove.

Problems Based on BPT and it's Converse

Question-1. If 'D' and 'E' are points on the respective sides AB and AC of ΔABC, such that AD = 6 cm, BD = 9 cm, AE = 8 cm, EC = 12 cm. prove that DE ∥ BC.

(CBSE-2015)

Proof:- In ΔABC



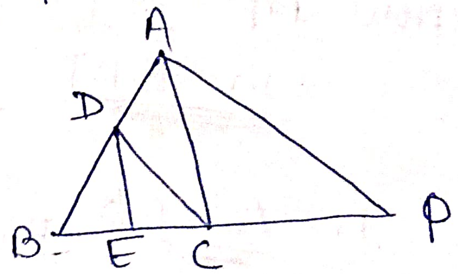
$$\frac{AD}{BD} = \frac{\cancel{6}^{2}}{\cancel{9}_{3}} = \frac{2}{3}$$

and $$\frac{AE}{EC} = \frac{\cancel{8}^{2}}{\cancel{12}_{3}} = \frac{2}{3}$$

Here $\boxed{\frac{AD}{BD} = \frac{AE}{EC}}$ Hence, DE ∥ BC proved (By B.P.T).

Question-2 In the given figure of ΔABC, DE ∥ AC. If DC ∥ AP, where point P lies on BC produced, then prove that $\frac{BE}{EC} = \frac{BC}{CP}$ [CBSE 2020] [CBSE 2015]

- Standard -



In ΔABC

given DE ∥ AC

By Thale's theorm.

$$\frac{BD}{AD} = \frac{BE}{EC} \quad \text{--- (1)}$$

Now in Δ ABP, given DC ∥ AP

By B.P.T (Thale's Theorem)

$$\frac{BD}{AD} = \frac{BC}{CP} \quad \text{--- (2)}$$

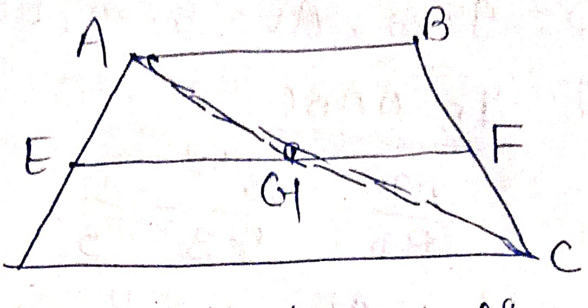
from (1) & (2)

$$\boxed{\frac{BE}{EC} = \frac{BC}{CP}}$$ proved.

Question:-3 ABCD is a trapezium with AB ∥ DC. E and F are two points on Non-parallel sides AD and BC respectively, such that EF parallel to AB.

Show that

$$\boxed{\frac{AE}{ED} = \frac{BF}{FC}}$$



or,

⇒ In a trapezium, show that any line draw parallel to the parallel sides of the trapezium, divides the non-parallel sides proportionally.

[CBSE-2011]

proof- Construction

Divide trapezium into two parts By joining point A and C. AC line Intersect EF at G.

Now In ΔACD, $EG \parallel DC$

By B.P.T $\frac{AE}{ED} = \frac{AG}{GC}$ —— (1)

In ΔACB $FG \parallel AB$

By B.P.T

$$\frac{GC}{AG} = \frac{CF}{BF}$$

By reciprocal

$$\frac{AG}{GC} = \frac{BF}{FC} \quad \text{—— (2)}$$

from eqn (1) & eqn (2)

$$\boxed{\frac{AE}{ED} = \frac{BF}{FC}}$$ proved.