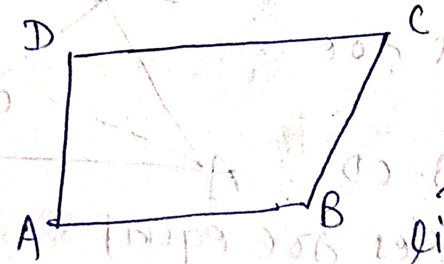
Quadrilaterals
(Revision Notes)

Class-09th
Lecture-01

Syllabus

1. Introduction, Types, Properties
2. (Prove) The diagonal divides a parallelogram into two congruent triangles.
3. (motivate) In a parallelogram, opposite sides are equal and conversely.
4. (motivate) In a parallelogram, opposite angles are equal and conversely.
5. (motivate) A quadrilateral is a parallelogram, the diagonals bisects each other and conversely.
6. (motivate) In a Δ, the line segment joining the mid-points of any two sides is parallel to the third side and its converse. (mid-point theorem).

Introduction



Definition:
A plane figure which is bounded by four line segment is called Quadrilateral.

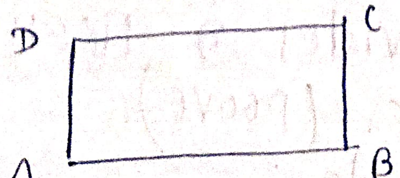
1. four line segment
   AB, BC, CD & AD
2. four vertices, A, B, C & D.
3. four Angles, $\angle A, \angle B, \angle C$ & $\angle D$.

Types of Quadrilateral

1. Square.

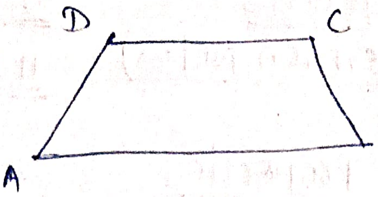
All sides & Angles are equal.
$AB = BC = CD = AD$.

2. Rectangle.



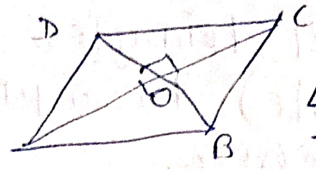
opposite sides are equal.
$AB = CD$, $BC = AD$.
$\angle A = \angle B = \angle C = \angle D = 90^\circ$.

(3.) Trepezium:-
Trepezium:-



AB || CD.
& AD, BC is
Non-parallel sides

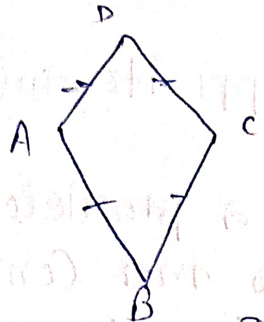
(4.) Rhombus :- All sides are equal & Diagonal Bisects at 90°.



AB = BC = CD = AD

$\angle AOB = 90°$

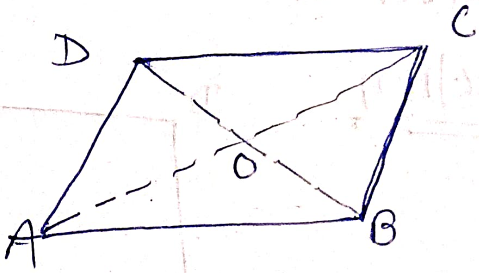
(5) Kite



AD = CD & AB = BC

\# Parallelogram :- In a quadrilateral both pair of opposite sides are equal & parallel to each other.

\# Properties of ||gram.



(1.) opposite sides are equal.
AD = BC, AB = CD

(2.) opposite angles are equal.
$\angle A = \angle C$, $\angle B = \angle D$.

(3.) Diagonals are bisected to each other.
OA = OC, OB = OD.

(4) pair of opposite sides are parallel to each other

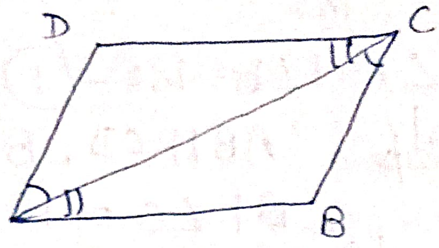
(5) Sum of adjacents angles are equal to 180°

$\angle A + \angle B = 180$, | $\angle A + \angle D = 180°$
$\angle B + \angle C = 180$ | $\angle C + \angle D = 180°$.

(6.) The diagonal divides a ||gram into two congruent triangle. (proove)
Remaining = motivate.

Theorem-1 A diagonal of a ||gram divides it into two congruent triangles.

Given: ABCD is a ||gram.
And AC is diagonal.



To prove:- $\Delta ABC \cong \Delta CDA$

Proof: Here ABCD is a ||gram. i.e. $AB \parallel CD$, $BC \parallel AD$.

In ΔABC and ΔCDA

$\angle BAC = \angle DCA$ (AB||CD and AC is transversal, so, Alternate Interior $\angle$s are equal)

$AC = CA$ (common side)

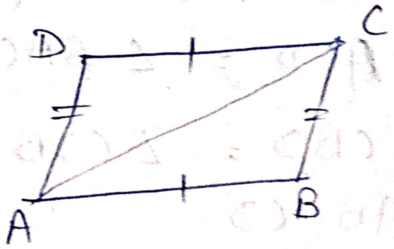
$\angle BCA = \angle DAC$ (BC||AD, AC is transversal) ∵ Alternate Interior angles are equal

By ASA congruency.

$\Delta ABC \cong \Delta CDA$ proved

Theorem-2 In a ||gram, opposite sides are equal.

Given- ABCD is a ||gram



To prove- $AB = CD$,
$AD = BC$

Proof: we know that the diagonal divides into two congruent Δ.
Here $\Delta ABC \cong ADC$

By c.p.c.t.

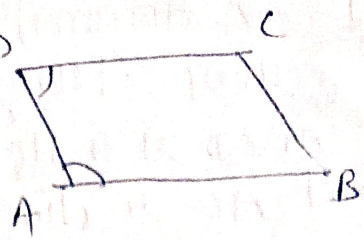
$AB = CD$, $AD = BC$ proved.

Theorem-3. In a ||gram, opposite sides are equal

Given ABCD is a ||gram,

To prove, $\angle A = \angle C$, $\angle B = \angle D$

AD ∥ BC, AB is transversal.
we know sum of consecutive angle = 180°



$\angle A + \angle B = 180°$ — (1)

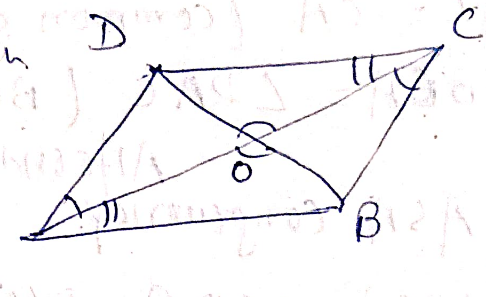
Similarly, AB ∥ CD, BC is transversal.

$\angle B + \angle C = 180°$ — (2)

from (1) & (2) $\angle A + \angle B = \angle B + \angle C$

$\therefore \angle A = \angle C$ prove.

7) Theorem - The diagonals of a parallelogram bisects each other.



Given ABCD is a parallelogram AC and BD is Diagonal.

To prove - OA = OC,
OB = OD

In Δ AOB and Δ COD.

~~$\angle AOB = \angle COD$ (vertically opposite angles are equal)~~

$\angle ACD = \angle BAC$ (Alternate Interior ∠)

$\angle CBD = \angle CDB$ (Alternate Interior angle)

AB = CD

BY ASA congruency,

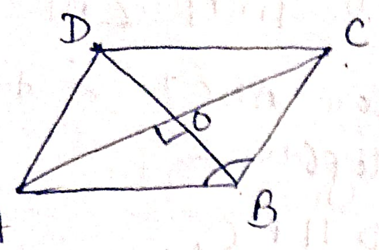
$\Delta AOB \cong \Delta COD$

BY C.P.C.T OA = OC & OB = OD proved.

Hence Diagonal Bisects to each other.

~~Prope~~ Problem Based on properties of parallelogram.

 # Question In the following figure, ABCD is a rhombus. If $\angle ABC = 68^\circ$, then ~~deamine~~/determine $\angle ACD$



Given, ABCD is a rhombus.
It's opposite sides are equal & parallel
Hence, ABCD is a parallelogram.
We, Sum of adjacent angle $= 180^\circ$

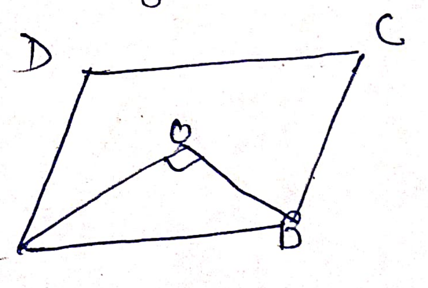
$\angle B + \angle C = 180^\circ$ $\therefore \angle C = 180^\circ - 68^\circ$

$\Rightarrow 68^\circ + \angle C = 180^\circ$ $= 112^\circ$

$\therefore \angle ACD = \frac{1}{2} \times \angle C = \cancel{\frac{1}{2} \times 112}^\circ = 56^\circ$ Ans

 # Question In a ||gram.
Show that the angle bisectors of two adjacent angles intersect at right angles.

Given: ABCD is a ||gram



We know,
Sum of adjacent angle is 180°

$\angle A + \angle B = 180^\circ$

$\Rightarrow \frac{1}{2}\angle A + \frac{1}{2}\angle B = 90^\circ$ — (1)

In ΔAOB.

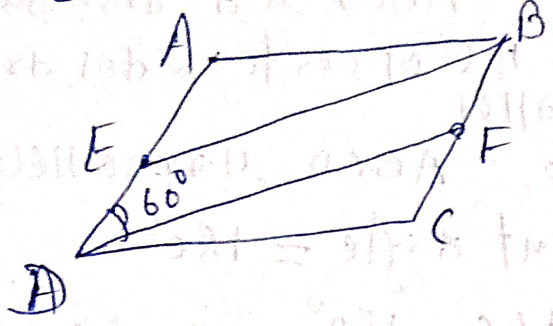
$\angle \frac{1}{2}\angle A + \frac{1}{2} B + \angle AOB = 180^\circ$

$90^\circ + \angle AOB = 180^\circ$

$\therefore \angle AOB = 90^\circ$ Ans

Question

In the given figure, ABCD is a parallelogram, E and F are points on opposite sides AD and BC respectively, such that $ED = \frac{1}{2} AD$ and $BF = \frac{1}{2} BC$. If $\angle ADF = 60^\circ$, then find $\angle BFD$.



Given ABCD is a parallelogram –

$AD \parallel BC$,

$AD \parallel BF$

Sum of Adjacent angle = 180°

$\Rightarrow 60^\circ + \angle BFD = 180$

$\therefore \angle BFD = 180 - 60 = 120^\circ$ Ans

Question.

#Quadrilaterals#

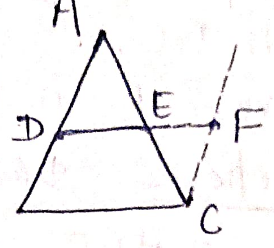
#class- 09th
Lecture-02

#Syllabus.

(1) (motivate) mid-point theorem

Statement:- The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.

Given:- In a $\triangle ABC$, D and E are the mid-points of AB and AC respectively.



To prove:- $DE \parallel BC$,

$DE = \frac{1}{2} BC$

Construction:- Draw a parallel line CF which is parallel to AB. Join EF.

Now In $\triangle ADE$ and $\triangle CFE$.

$\angle AED = \angle CEF$ (vertically opposite angle)

$AE = CE$ (E is the mid-point on the side AC)

$\angle A = \angle C$ (Alternate Interior angle, $AB \parallel CF$)

By ASA congruency.

$\triangle AED \cong \triangle CEF$

By C.P.C.T $AD = CF$ and $DE = EF$ —(1)

Also $AD = BD$ (D is mid-point) —(2)

from (1) & (2)

$AD = BD = CF$ & $BD \parallel CF$ (By construction)

Hence, BCFD is a llgram.

$DF \parallel BC$ (pair of opposite sides are parallel)

$\therefore$ $DE \parallel BC$ proved.

and $DF = BC$

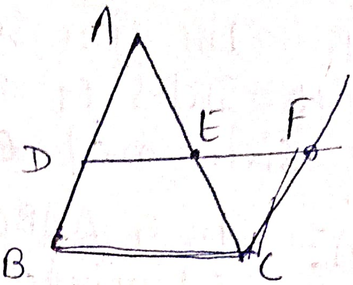
$\Rightarrow$ $DE + EF = BC$

$\Rightarrow$ $2DE = BC$

$\therefore$ $\boxed{DE = \frac{1}{2} BC}$ proved.

(Converse of mid-point Theorem):-

The line drawn through the mid-point of one side of a triangle, parallel to another side bisect the third-side.



Given- In ΔABC, D is the mid-point of AB and $DE \parallel BC$.

To prove- E is the mid-point.

Construction- Draw CF line parallel to AB. join EF

In ΔADE and ΔCEF

$\angle A = \angle C$ ($AB \parallel CF$, AC is transversal)

$\angle ADE = \angle CFE$ (Alternate Interior).

$AD = CF$ (BCFD is ||gram).

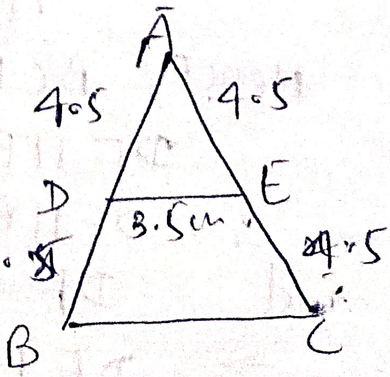
By ASA congruency.

$\Delta ADE \cong \Delta CEF$.

By C.P.C.T $\boxed{AE = CE}$

it will be possible of E is the mid-point.

Example #1 question ΔABC is an isosceles Δ in which $AB = AC$. D and E are the mid-points of sides AB and AC and $DE = 3.5$ cm. find the perimeter of ΔABC, when $AD = 4.5$ cm.



By mid-point theorem.

$AB = AC = 9$ cm

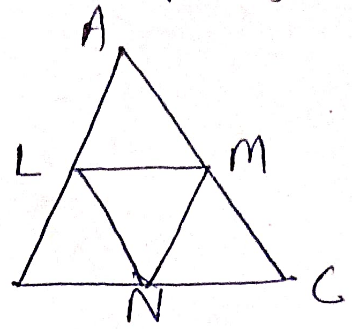
$DE = \frac{1}{2} BC$

$3.5 = \frac{1}{2} BC \quad \therefore BC = 7$ cm

perimeter:- $AB + BC + AC$

$9 + 7 + 9 = 25$ cm

ABC is an equilateral triangle and L, M and N are the mid-points of the sides AB, BC and CA respectively. Prove that ΔLMN is an equilateral triangle.



Given. ΔABC is an equilateral.

$AB = BC = CA$

To prove- ΔLMN is an equilateral triangle.

We know that

By mid point theorem.

$LM = \frac{1}{2} BC$ — (1)

$MN = \frac{1}{2} AB$ — (2)

$LN = \frac{1}{2} AC$ — (3)

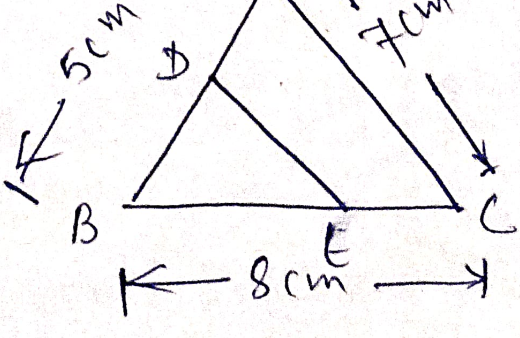
from (1) & (2)

$LM = MN = LN$ $[AB = BC = AC]$

Hence ΔLMN is an equilateral Δ.

In ΔABC, $AB = 5cm$, $BC = 8cm$ and $AC = 7cm$. If D and E are respectively the mid-point of AB and BC, determine the length of DE.

By mid-point theorem



$DE = \frac{1}{2} AC$

$DE = \frac{1}{2} \times 7 = 3.5cm$