

Polynomials

(Class - 9th)

①

Combination of constant and variable.
 ⇒ Power on variables be Non-Negative Integers.

Example -

$$P(x) = ax^2 + bx + c$$

Here in x^m power on $x = 2$

So, called two degree polynomial.

& this having 3-term, so called trinomial.

Zero of polynomial

Let $P(x)$ be a polynomial. If $P(x) = 0$, then we say that x is a zero of the polynomial $P(x)$.

Note:- finding the zeros of polynomial $P(x)$ means solving the equation $P(x) = 0$.

Factor Theorem

Let $f(x)$ be a polynomial of degree $n > 1$ and let α' be any real number.

(i) If $f(\alpha) = 0$ then $(x-\alpha)$ is a factor of $f(x)$.

(ii) If $(x-\alpha)$ is a factor of $f(x)$ then $f(\alpha) = 0$.

Factorisation / Based on following formula

$$1. (a+b)^2 = a^2 + b^2 + 2ab$$

$$2. (a-b)^2 = a^2 + b^2 - 2ab$$

$$3. (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca.$$

$$④ (a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

$$⑤ (a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

$$⑥ a^2 - b^2 = (a+b)(a-b)$$

$$⑦ a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$⑧ a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Important

$$① a^3 + b^3 + c^3 = 3abc \text{ if } a+b+c=0$$

$$② \text{ if } a+b+c \neq 0 \text{ then}$$

$$a^3 + b^3 + c^3 = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

Important factorisation

$$P(x) = x^3 + 13x^2 + 32x + 20$$

(By factorisation method.)

Solution (i) put $x=-1$ and check this is factor or not?

$$\begin{aligned} P(-1) &= (-1)^3 + 13(-1)^2 + 32(-1) + 20 \\ &= -1 + 13 - 32 + 20 \quad (x+1) \text{ is a factor.} \\ &= -33 + 33 = 0 \end{aligned}$$

$$P(x) = x^3 + 13x^2 + 32x + 20$$

$$P(x) = x^3 + x^2 + 12x^2 + 12x + 20x + 20$$

$$= x^2(x+1) + 12x(x+1) + 20(x+1)$$

$$= (x+1)(x^2 + 12x + 20)$$

$$= (x+1) \left\{ x^2 + 10x + 2x + 20 \right\}$$

$$= (x+1) \left\{ x(x+10) + 2(x+10) \right\}$$

$$= (x+1)(x+10)(x+2) \text{ factors.}$$